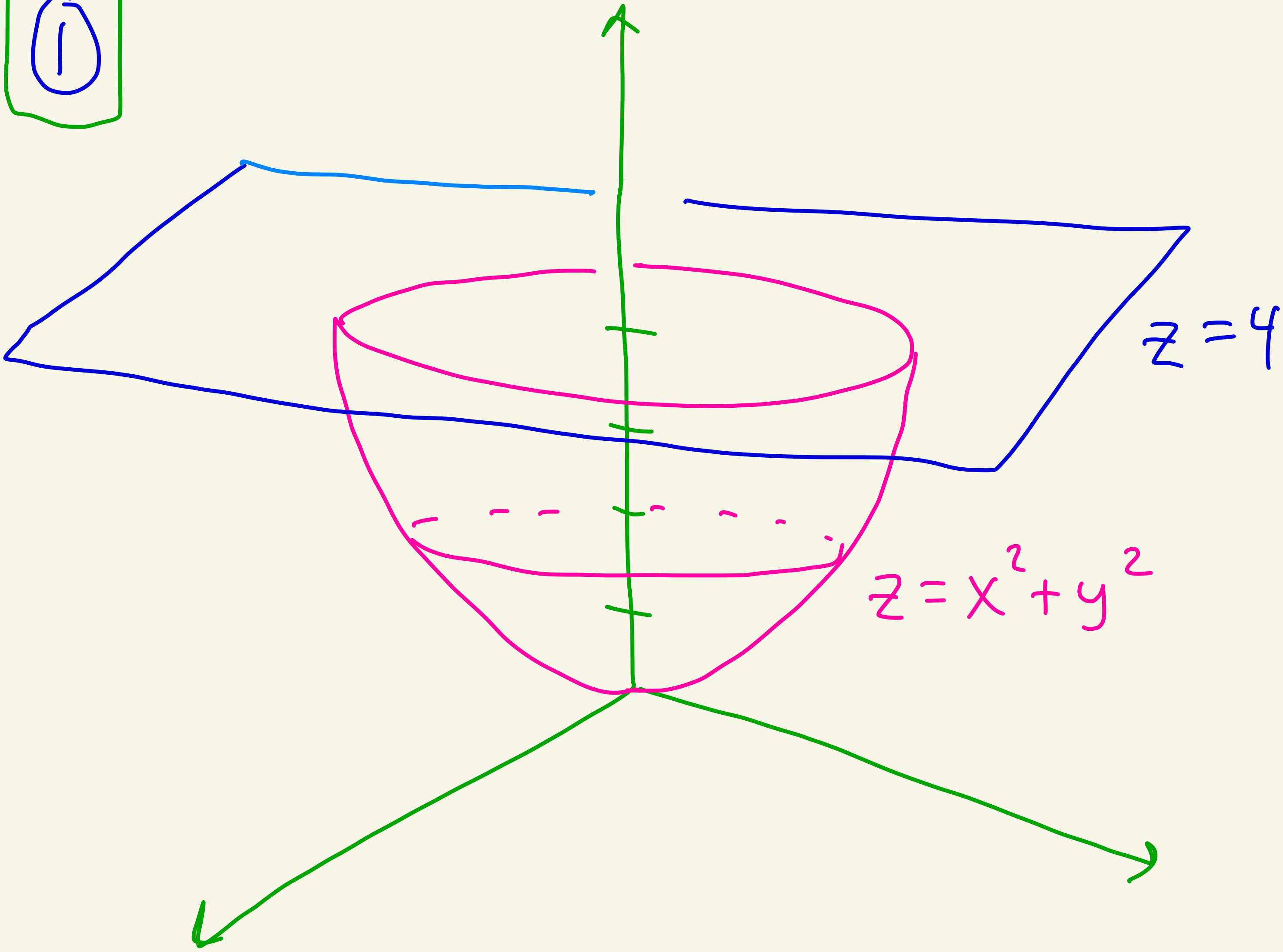
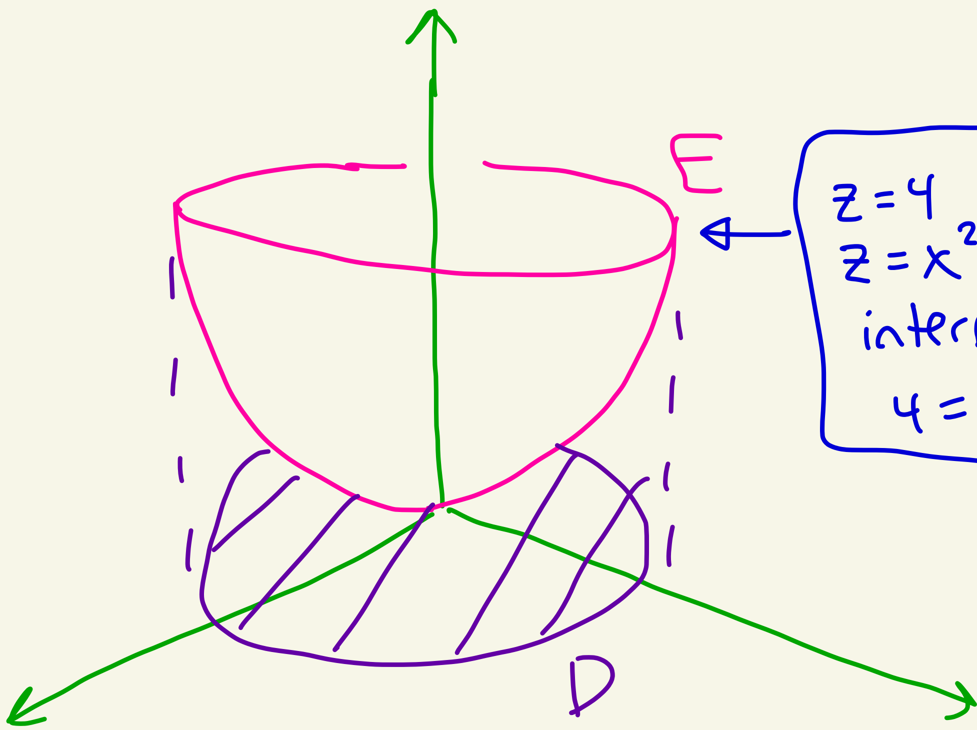
Math 2130

HW 6 Solutions

1



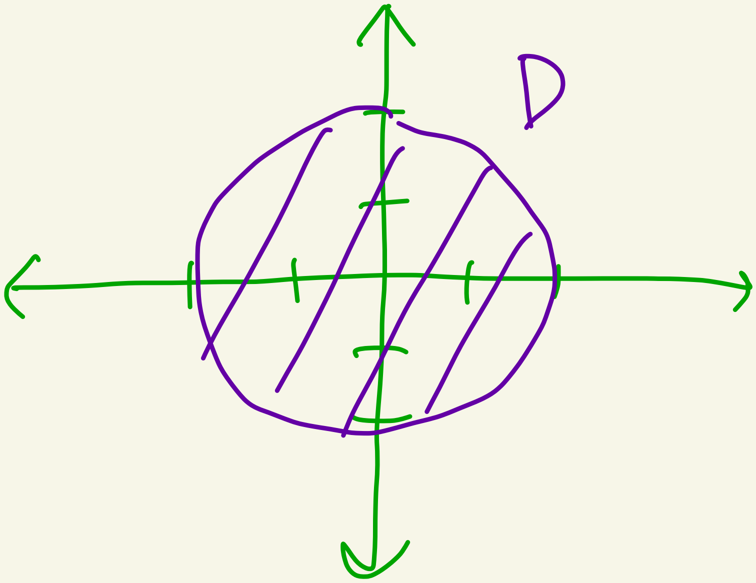


$z = 4$

$z = x^2 + y^2$

intersect at

$4 = x^2 + y^2$



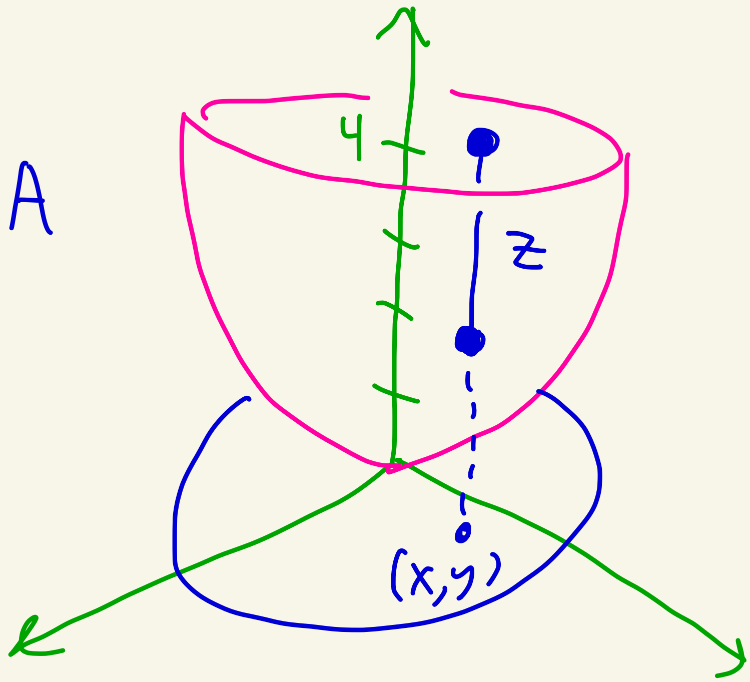
D is given by:

$x^2+y^2 \le 4$

$0 \le \theta \le 2\pi$

$0 \le r \le 2$

$$\text{Volume of } E = \iiint_E 1 \, dV$$



$$= \iint_D \left[\int_{x^2+y^2}^{4} 1 \, dz \right] dA$$

$$= \iint_D \left[z \Big|_{x^2+y^2}^{4} \right] dA$$

$$= \iint_D \left[4 - (x^2+y^2) \right] dA$$

$0 \le \theta \le 2\pi$

$0 \le r \le 2$

$r^2 = x^2 + y^2$

$$= \int_0^{2\pi} \int_0^{2} \left[4 - r^2\right] r \, dr \, d\theta$$ $dA = r\,dr\,d\theta$

$$= \int_0^{2\pi} \int_0^{2} \left(4r - r^3\right) dr \, d\theta$$

$$= \int_0^{2\pi} \left(2r^2 - \frac{1}{4} r^4 \Big|_0^2 \right) d\theta$$

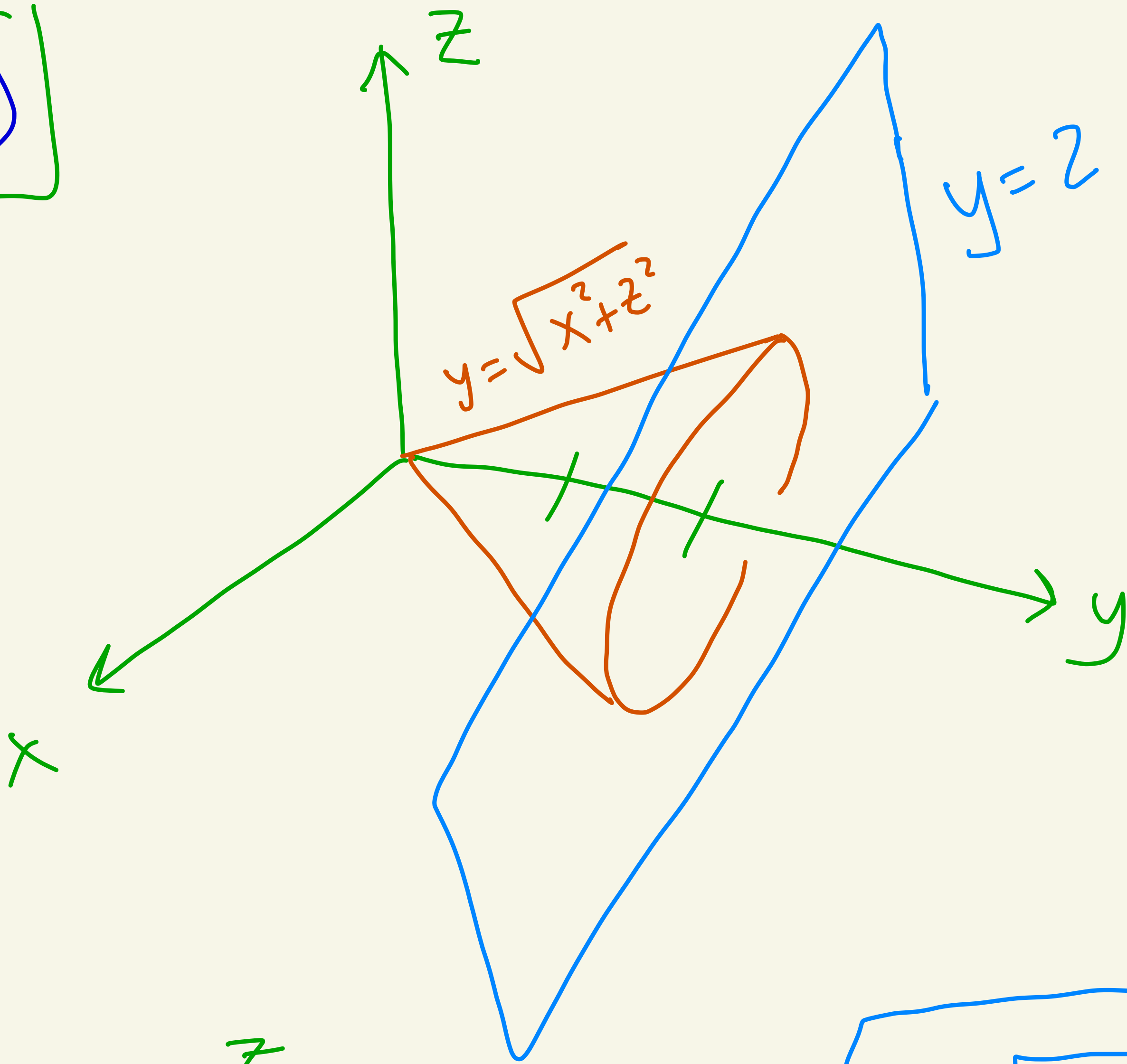
$$= \int_0^{2\pi} \left(2 \cdot 2^2 - \frac{1}{4} \cdot 2^4\right) - \left(0 - 0\right) \, d\theta$$

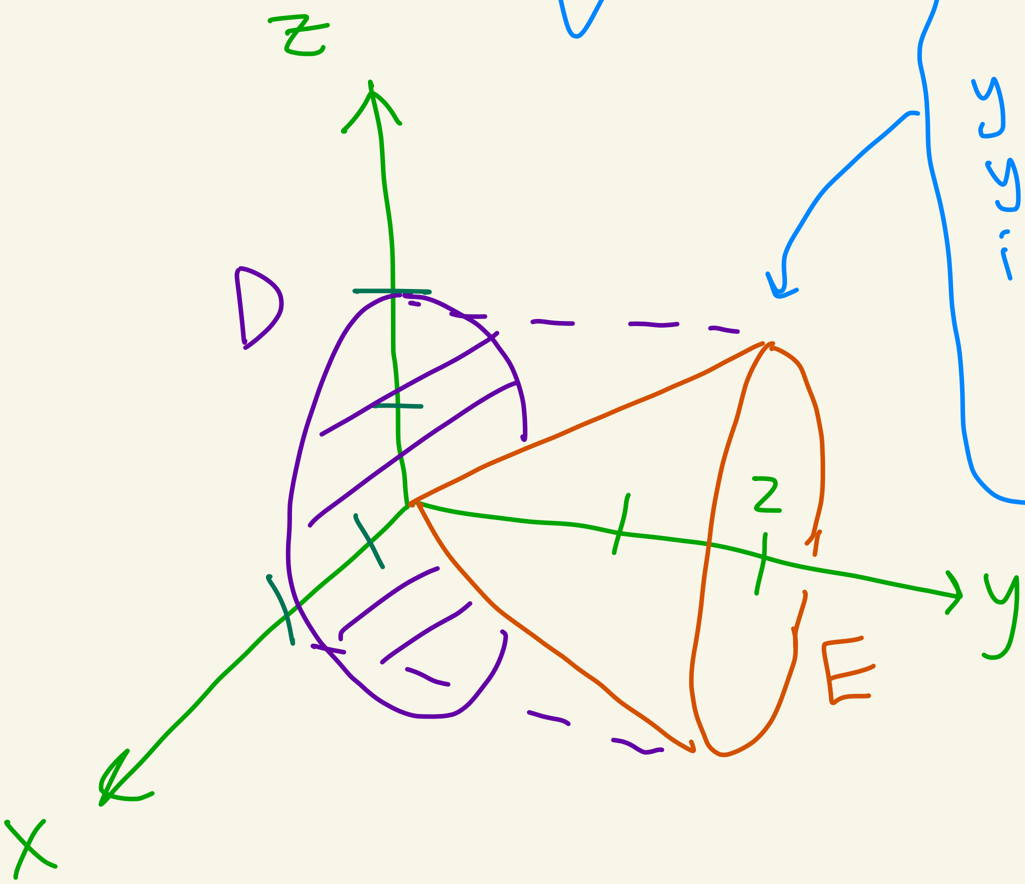
$$= \int_0^{2\pi} 2 \, d\theta$$

$$= 2\theta \Big|_0^{2\pi}$$

$$= 2\left(2\pi - 0\right) = \boxed{4\pi}$$

2





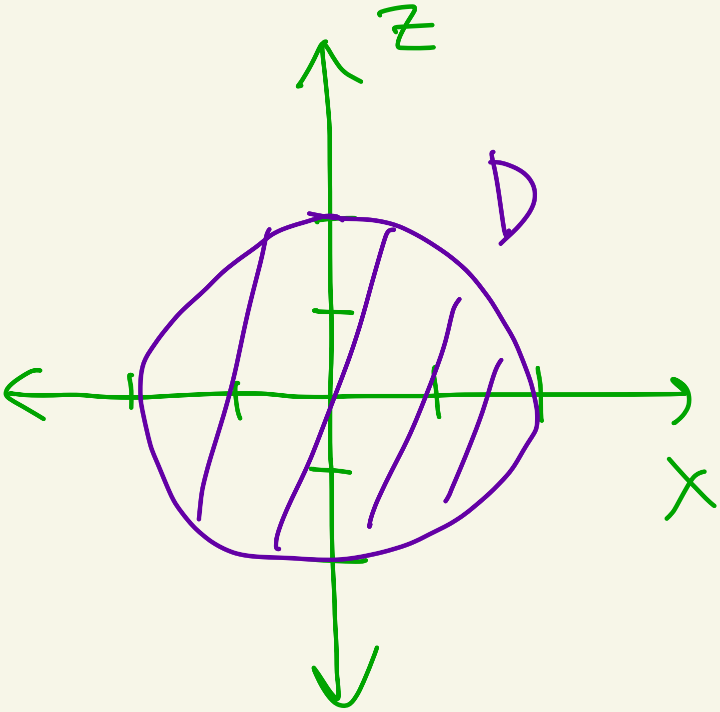
$y=\sqrt{x^2+z^2}$

$y=2$

intersect at:

$2=\sqrt{x^2+z^2}$

$4=x^2+z^2$



D is given by:

$x^2 + z^2 \le 4$

or

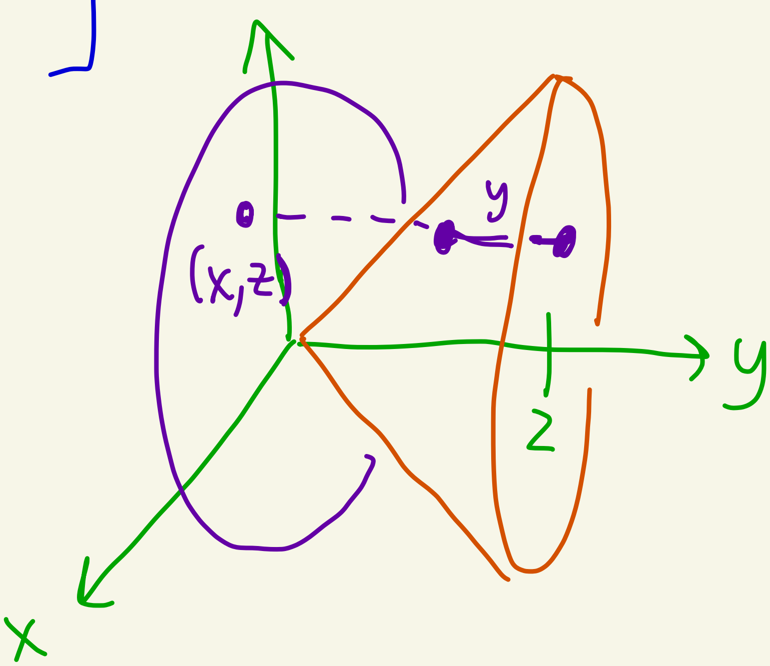
$0 \le \theta \le 2\pi$

$0 \le r \le 2$

$$\text{Volume of } E = \iiint_E 1 \, dV$$

$$= \iint_D \left[\int_{x^2+z^2}^{4} 1 \, dy \right] dA$$

$$= \iint_D \left[y \Big|_{\sqrt{x^2+z^2}}^{4} \right] dA$$



$$= \iint_D \left(4 - \sqrt{x^2+z^2}\right) dA$$

$0 \le \theta \le 2\pi$

$0 \le r \le 2$

$r^2 = x^2 + z^2$

$dA = r\,dr\,d\theta$

$$= \int_0^{2\pi} \int_0^2 \left(4 - \sqrt{r^2}\right) r\,dr\,d\theta$$

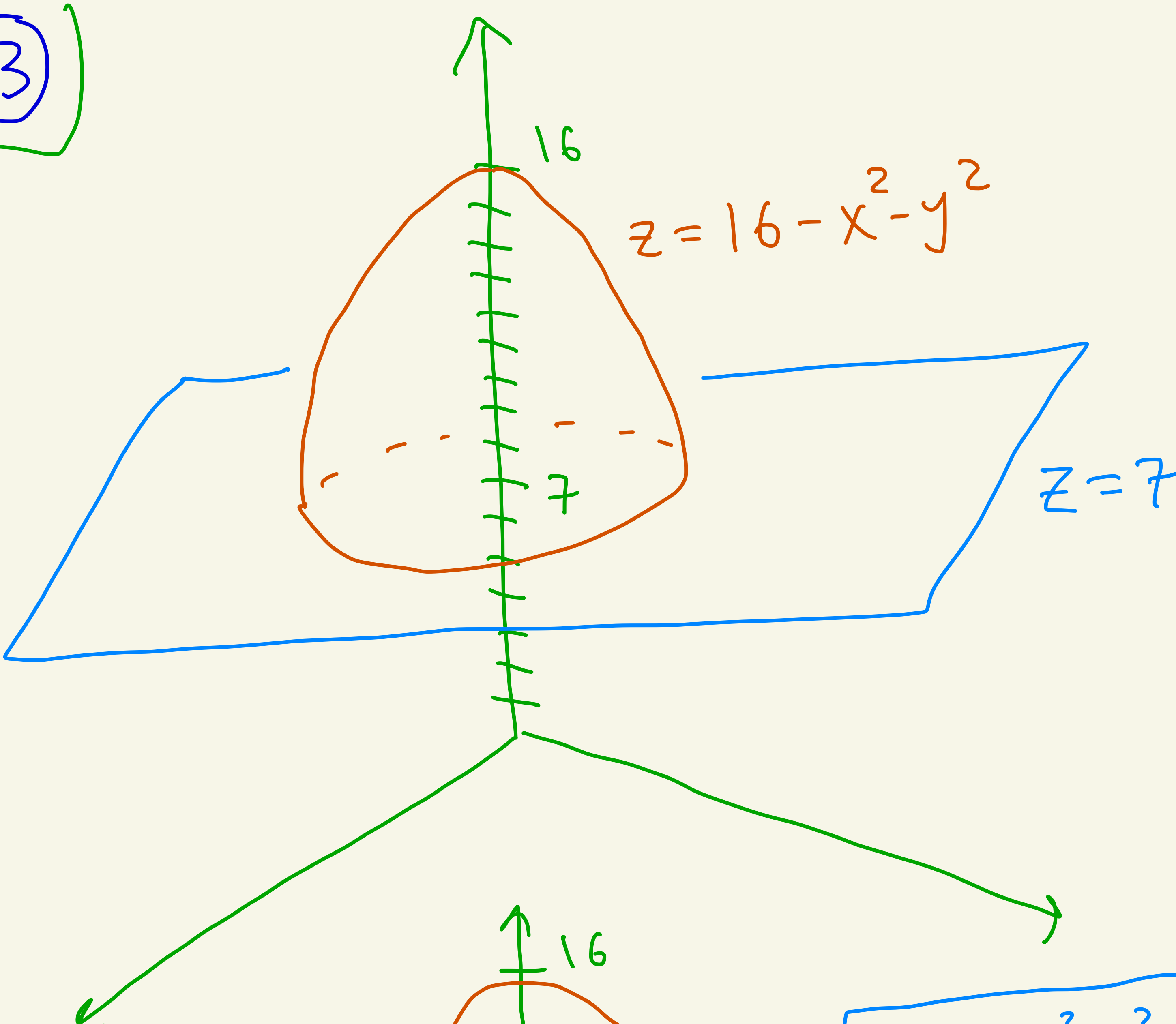
$$= \int_0^{2\pi} \int_0^2 \left(4r - r^2\right) dr\,d\theta$$

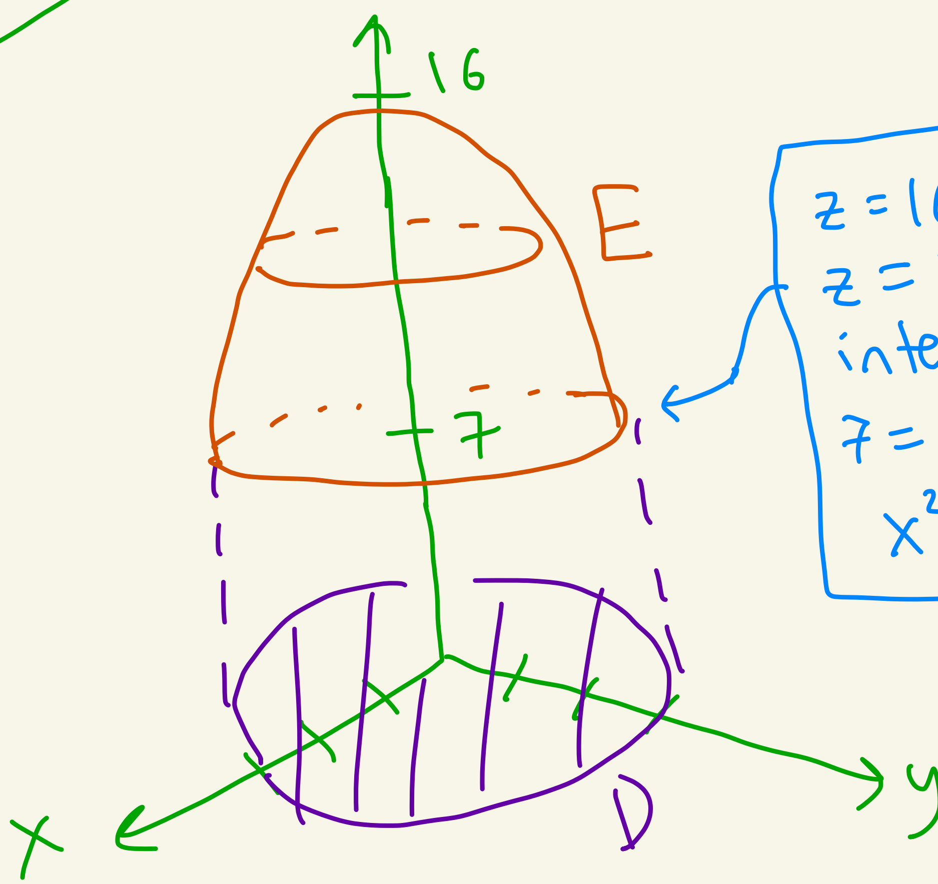
$$= \int_0^{2\pi} \left(2r^2 - \frac{1}{3}r^3 \Big|_0^2\right) d\theta$$

$$= \int_0^{2\pi} \left(\left(2 \cdot 2^2 - \frac{1}{3}2^3\right) - 0\right) d\theta = \int_0^{2\pi} \frac{16}{3}\,d\theta$$

$$= \frac{16}{3}\theta \Big|_0^{2\pi} = \frac{16}{3}(2\pi - 0) = \boxed{\frac{32}{3}\pi}$$

3





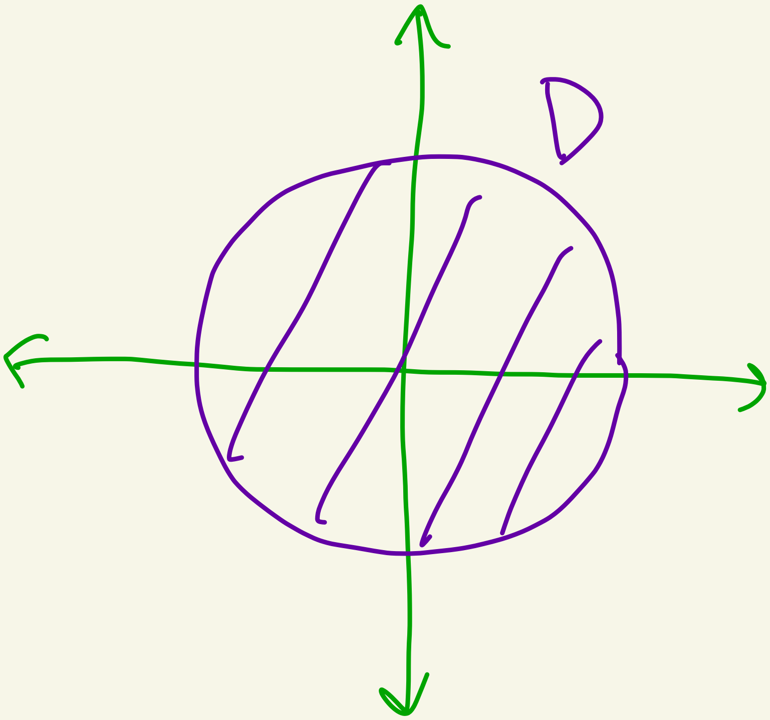
$z = 16 - x^2 - y^2$

$z = 7$

intersect at

$7 = 16 - x^2 - y^2$

$x^2 + y^2 = 9$



D is given by

$x^2 + y^2 \le 9$

or

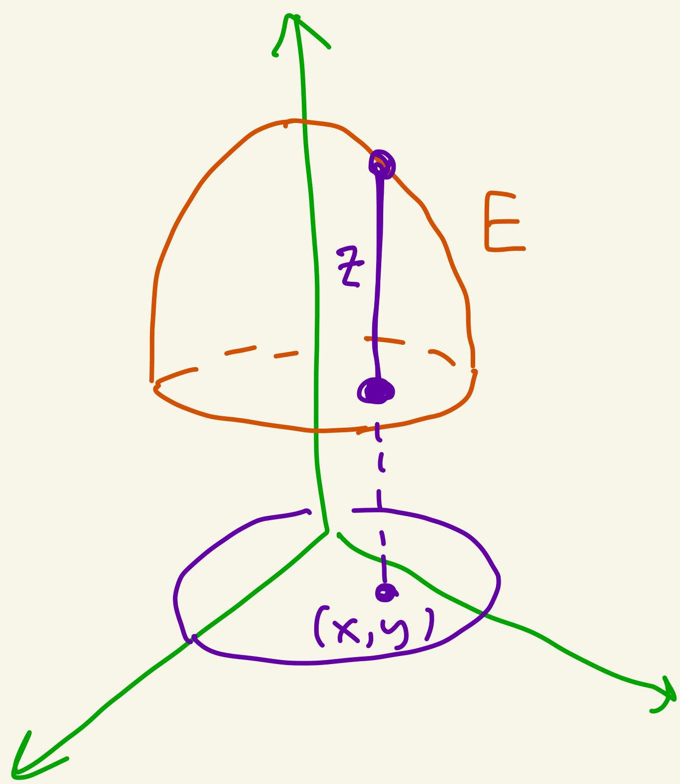
$0 \le \theta \le 2\pi$

$0 \le r \le 3$

$$\text{Volume of } E = \iiint_E 1 \, dV$$

$$= \iint_D \left[\int_7^{16-x^2-y^2} 1 \, dz \right] dA$$

$$= \iint_D \left[z \Big|_7^{16-x^2-y^2} \right] dA$$



$$= \iint_D \left[(16 - x^2 - y^2) - 7 \right] dA$$

$$= \iint_D \left[9 - x^2 - y^2 \right] dA$$

$0 \le \theta \le 2\pi$

$0 \le r \le 3$

$r^2 = x^2 + y^2$

$dA = r\,dr\,d\theta$

$$= \int_0^{2\pi} \int_0^3 \left[9 - r^2 \right] r\,dr\,d\theta$$

$$= \int_0^{2\pi} \int_0^3 \left[9r - r^3 \right] dr\,d\theta$$

$$= \int_0^{2\pi} \left[\frac{9}{2} r^2 - \frac{1}{4} r^4 \Big|_0^3 \right] d\theta$$

$$= \int_0^{2\pi} \left[\frac{9}{2} \cdot 3^2 - \frac{1}{4} \cdot 3^4 \right] - \left[0 - 0 \right] d\theta$$

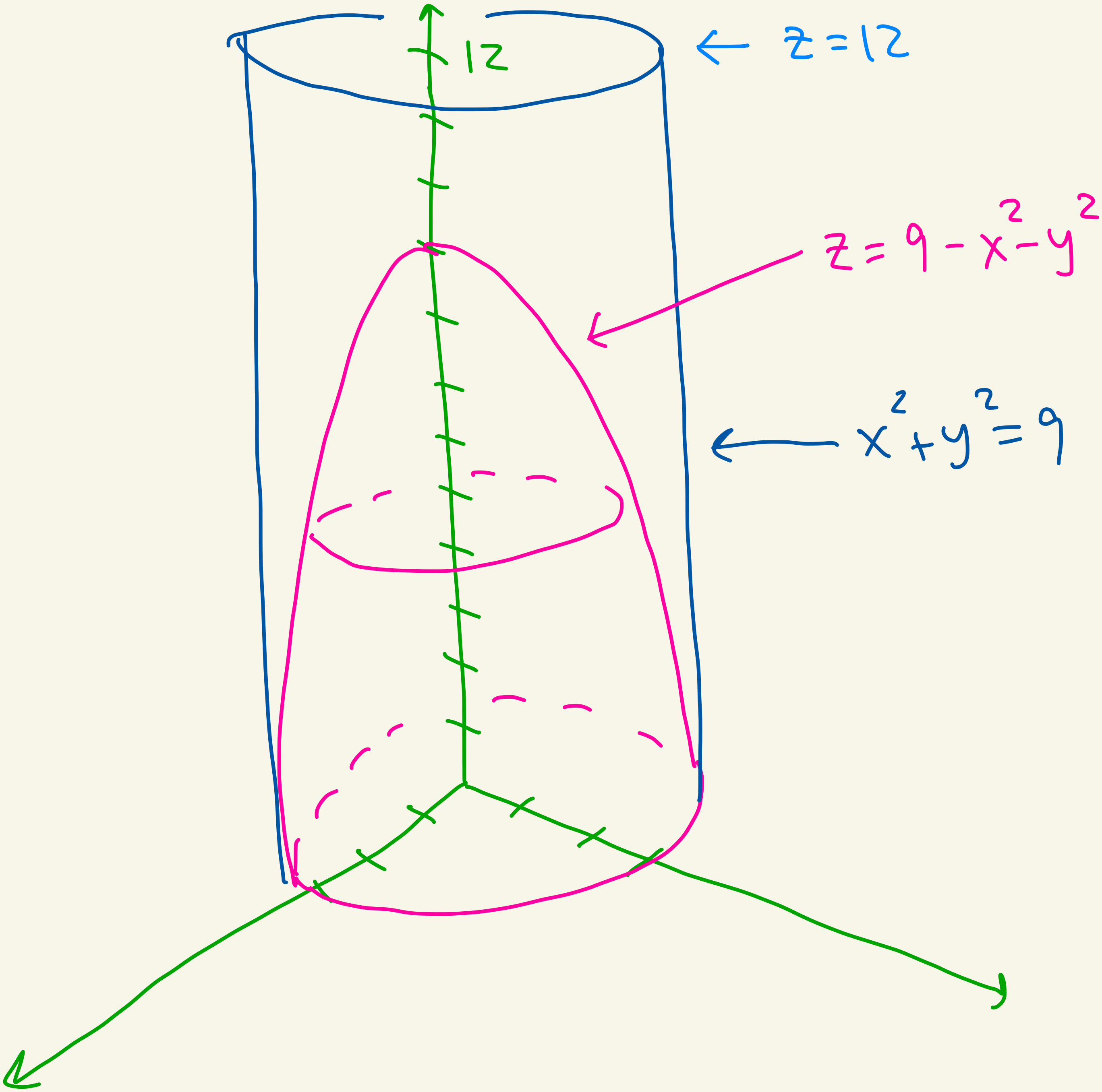
$$= \int_0^{2\pi} \frac{81}{4}\, d\theta$$

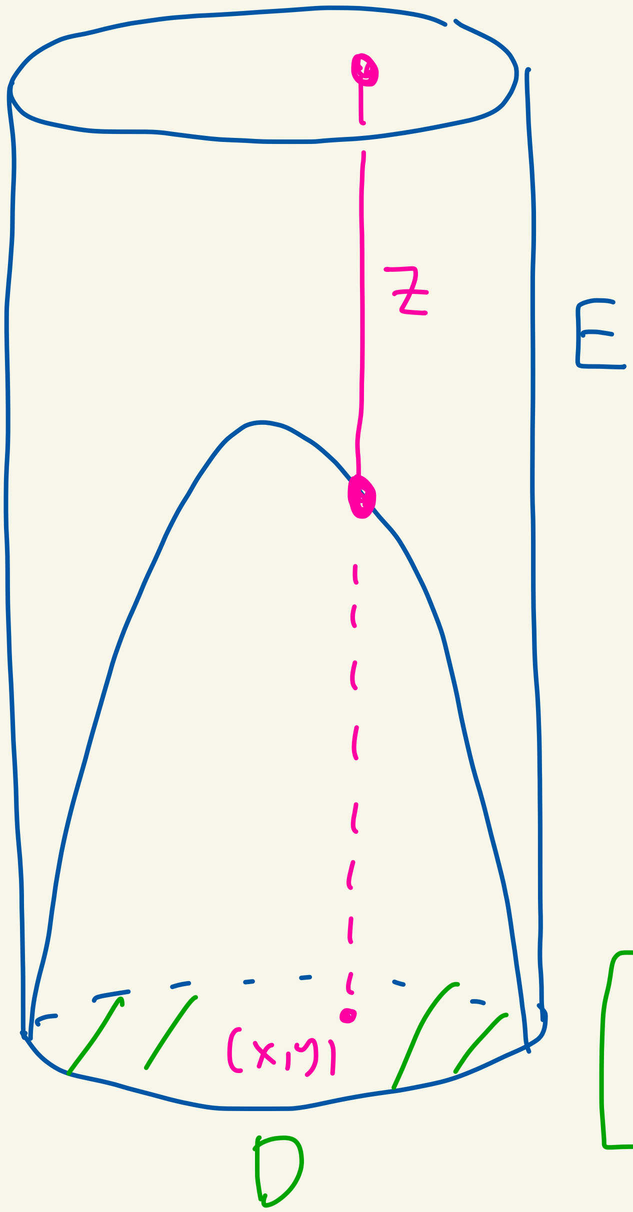
$$= \frac{81}{4}\theta \Big|_0^{2\pi}$$

$$= \frac{81}{4}(2\pi - 0)$$

$$= \boxed{\frac{81}{2}\pi}$$

4





D is $x^2+y^2 \le 9$

$0 \le \theta \le 2\pi$

$0 \le r \le 3$

$$\text{Volume of } E = \iiint_E 1$$

$$= \iint_D \left[\int_{9-x^2-y^2}^{12} 1 \, dz \right] dA$$

$$= \iint_D \left[z \Big|_{9-x^2-y^2}^{12} \right] dA$$

$$= \iint_D \left(12 - (9 - x^2 - y^2)\right) dA$$

$0 \le \theta \le 2\pi$
$0 \le r \le 3$
$r^2 = x^2 + y^2$
$dA = r\,dr\,d\theta$

$$= \int_0^{2\pi} \int_0^3 (3 + r^2)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^3 (3r + r^3)\, dr\, d\theta$$

$$= \int_0^{2\pi} \left(3\frac{r^2}{2} + \frac{r^4}{4}\right)\Big|_0^3 d\theta$$

$$= \int_0^{2\pi} \left(\frac{3}{2} \cdot 3^2 + \frac{1}{4} \cdot 3^4 \right) - (0 - 0) \, d\theta$$

$$= \int_0^{2\pi} \frac{135}{4} \, d\theta$$

$$= \frac{135}{4} \theta \Big|_0^{2\pi}$$

$$= \frac{135}{4} (2\pi - 0)$$

$$= \boxed{\frac{135}{2} \pi}$$

$\boxed{⑤}$

$$\text{mass of } E = \iiint_E (2-z)\,dV$$

$$= \int_0^3 \int_0^2 \int_0^1 (2-z)\,dz\,dy\,dx$$

$$= \int_0^3 \int_0^2 \left(2z - \frac{1}{2}z^2 \Big|_{z=0}^{1} \right) dy\,dx$$

$$= \int_0^3 \int_0^2 \left(2-\frac{1}{2}\right) - (0-0)\;dy\,dx$$

$$= \int_0^3 \int_0^2 \frac{3}{2}\, dy\, dz$$

$$= \int_0^3 \frac{3}{2} y \Big|_0^2 \, dz$$

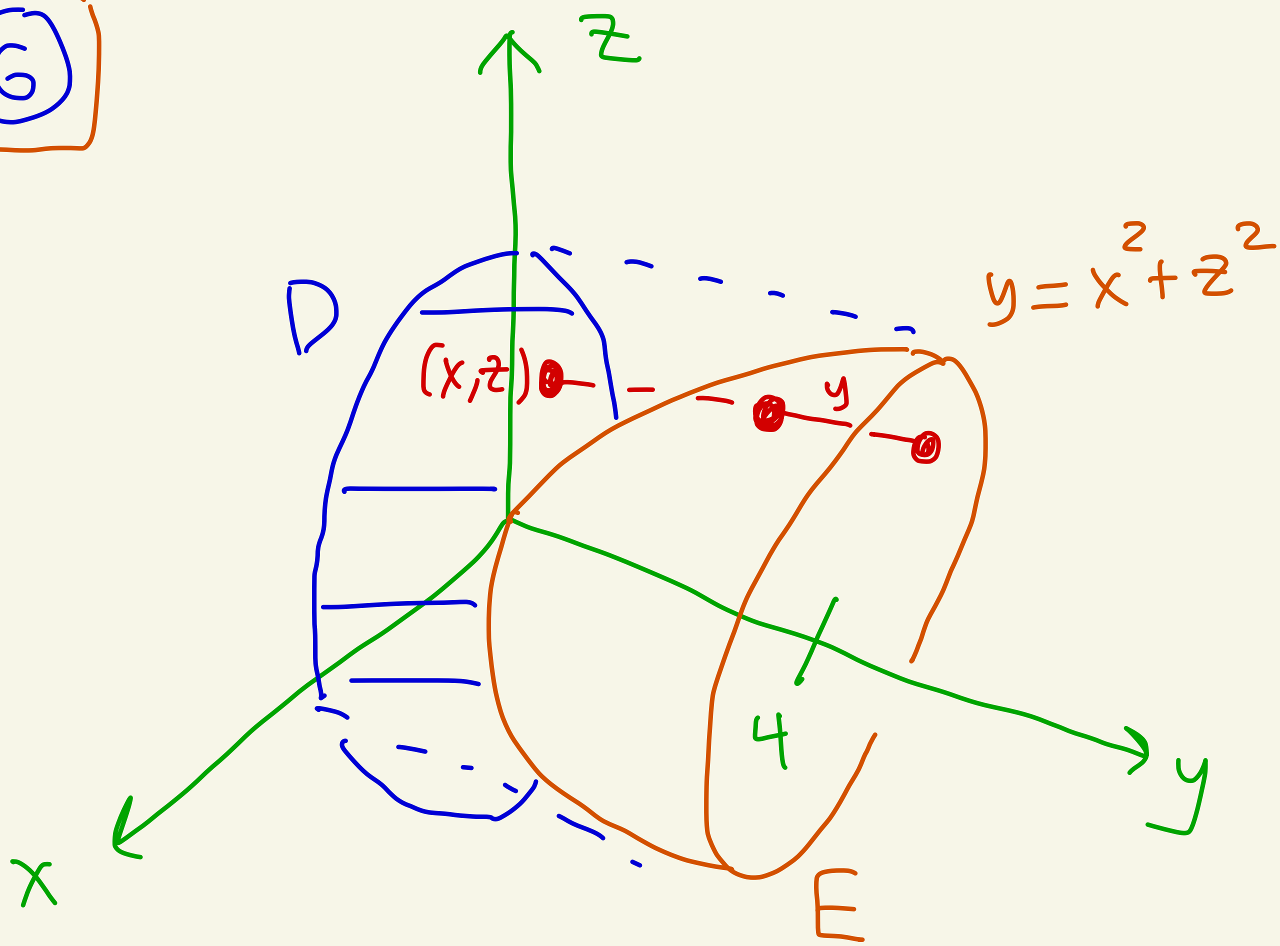
$$= \int_0^3 \frac{3}{2}(2-0)\, dz$$

$$= \int_0^3 3\, dz = 3z \Big|_0^3$$

$$= 3(3-0)$$

$$= \boxed{9}$$

6



$$\boxed{\begin{array}{l} D \text{ is } x^2+z^2 \le 4 \\ 0 \le \theta \le 2\pi \\ 0 \le r \le 2 \end{array}}$$

$$\text{mass of } E = \iiint_E \sqrt{x^2+z^2}\, dV$$

$$= \iint_D \left[\int_{x^2+z^2}^{4} \sqrt{x^2+z^2}\, dy \right] dV$$

$$= \iint_D \left(\sqrt{x^2+z^2}\right) y \Big|_{y=x^2+z^2}^{4} \, dV$$

$$= \iint_D \left[4\sqrt{x^2+z^2} - (x^2+z^2)\sqrt{x^2+z^2}\right] dV$$

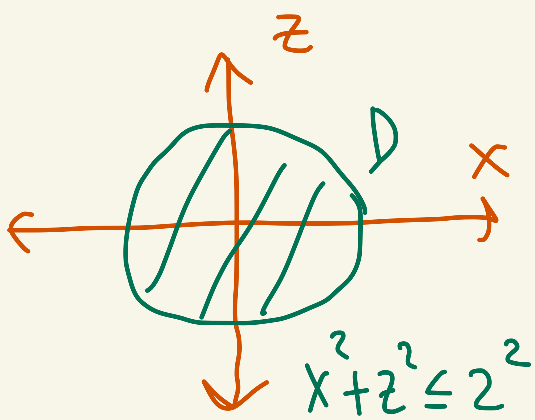
$$= \int_0^{2\pi}\int_0^{2} \left[4\sqrt{r^2} - r^2\sqrt{r^2}\right] r\,dr\,d\theta$$

$0 \le \theta \le 2\pi$

$0 \le r \le 2$

$r^2 = x^2 + z^2$

$dA = r\,dr\,d\theta$



$$= \int_0^{2\pi}\int_0^{2} \left[4r - r^2 \cdot r\right] r\,dr\,d\theta$$

$$= \int_0^{2\pi} \int_0^{2} \left(4r^2 - r^4\right) dr\, d\theta$$

$$= \int_0^{2\pi} \left(\frac{4}{3} r^3 - \frac{1}{5} r^5 \Big|_0^2 \right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{4}{3} \cdot 2^3 - \frac{1}{5} \cdot 2^5 \right) - \left(0 - 0\right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{160 - 96}{15} \right) d\theta$$

$$= \int_0^{2\pi} \frac{64}{15} d\theta = \frac{64}{15} \theta \Big|_0^{2\pi} = \frac{64}{15}\left(2\pi - 0\right)$$

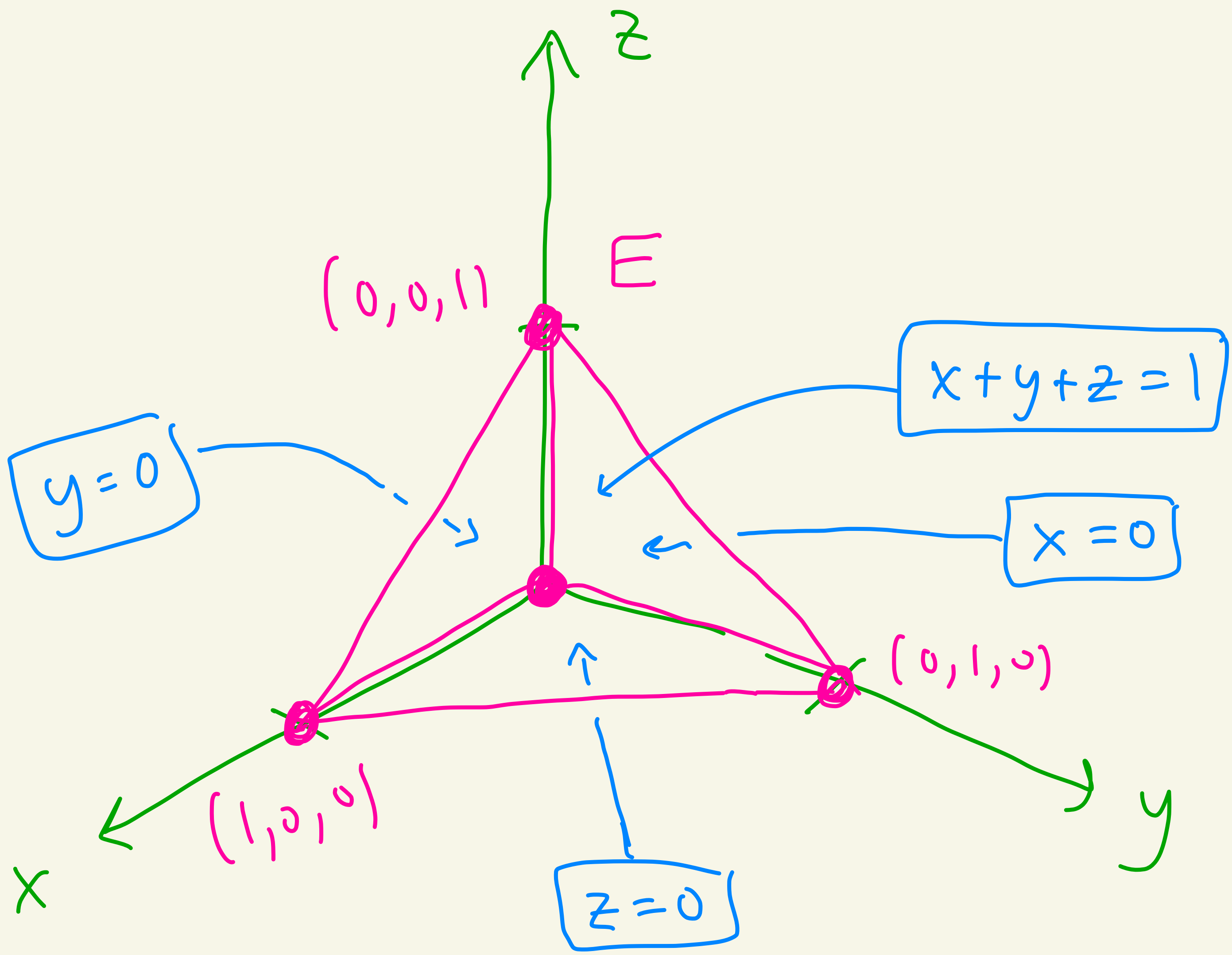
$$= \boxed{\frac{128}{15}\pi}$$

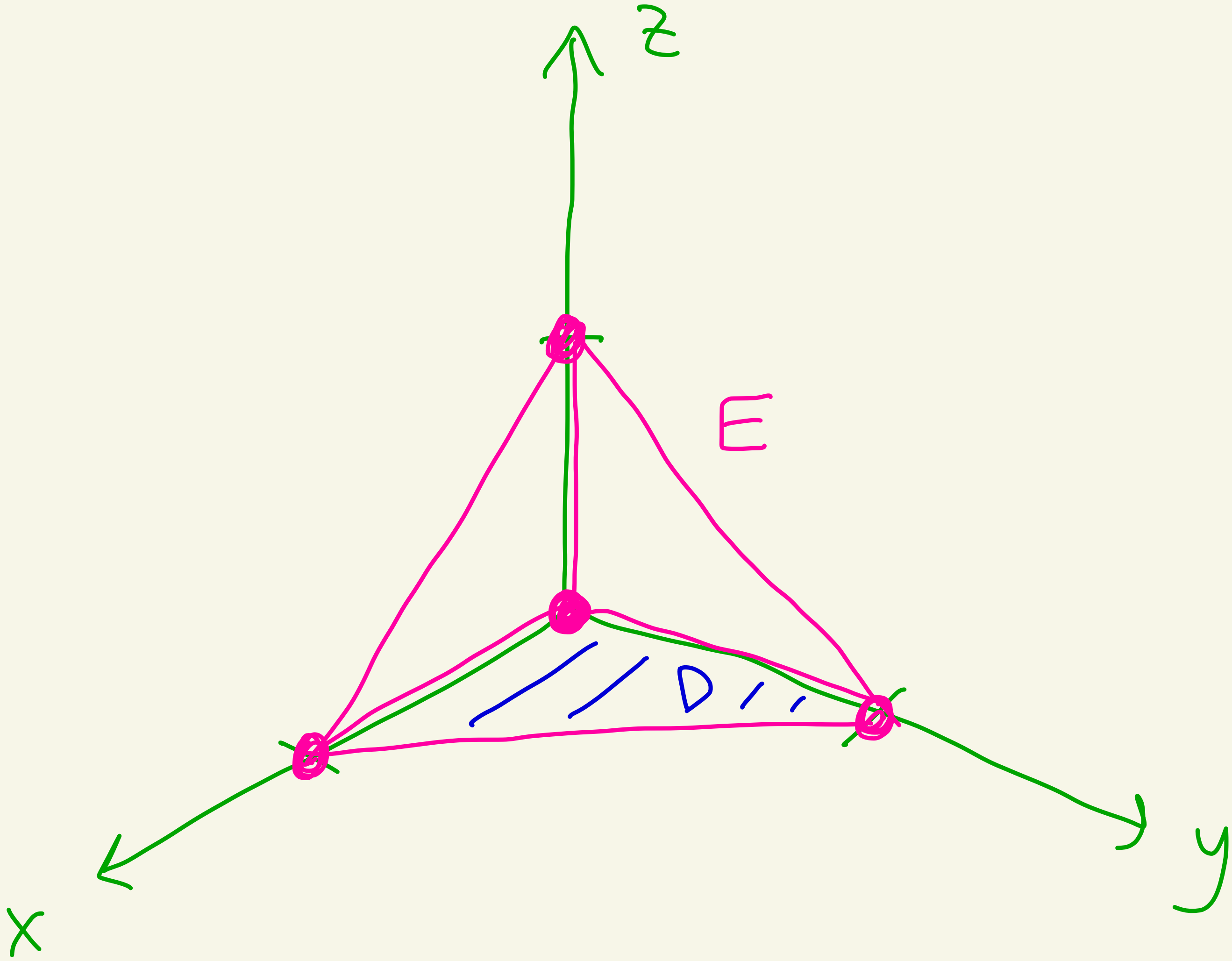
$\boxed{7}$ Some points on $x+y+z=1$ are:

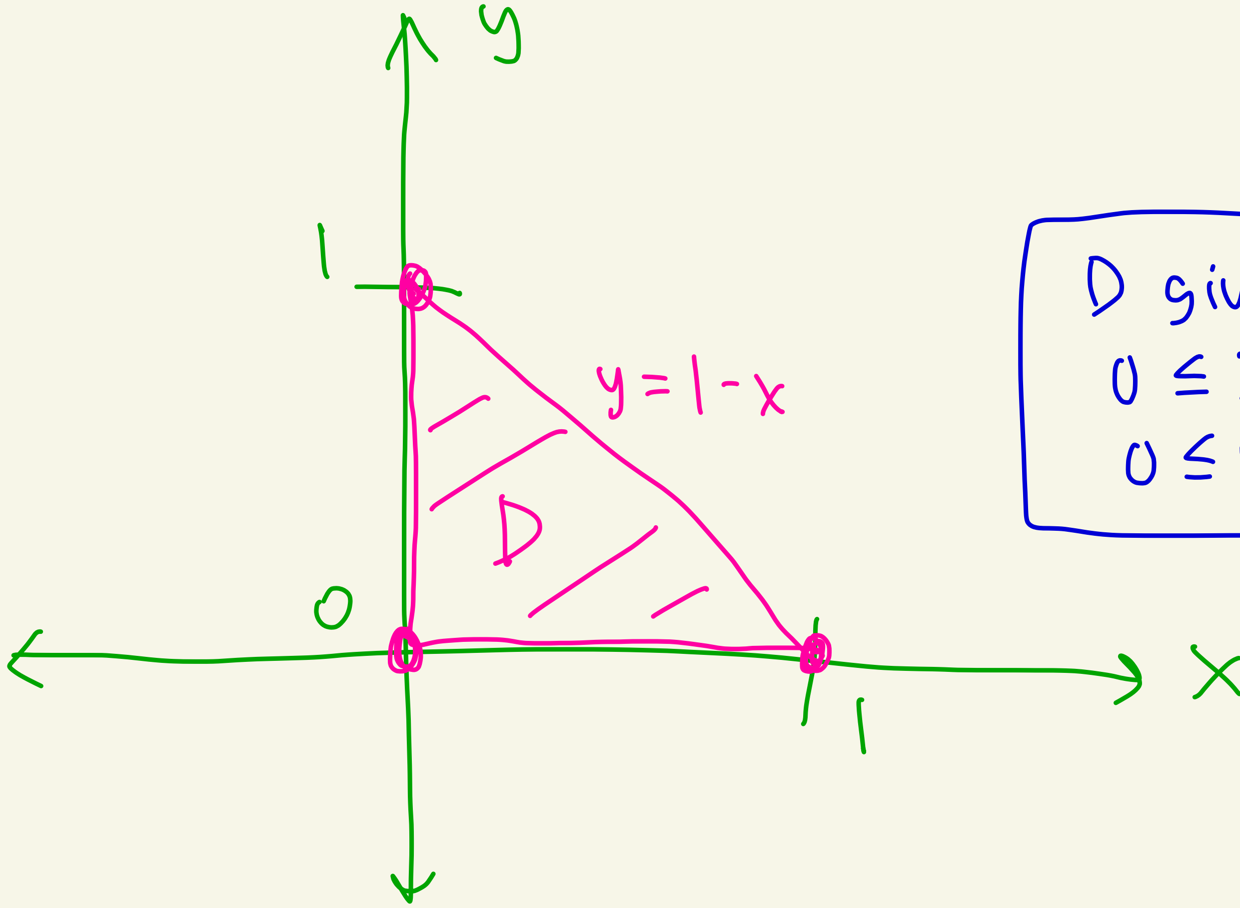
$(x,y,z)=(1,0,0)$ since $1+0+0=1$

$(x,y,z)=(0,1,0)$ since $0+1+0=1$

$(x,y,z)=(0,0,1)$ since $0+0+1=1$







D given by
$0 \le x \le 1$
$0 \le y \le 1 - x$

mass of $T = \iiint_T y \, dV$

$$= \iint_D \left[\int_0^{1-x-y} y \, dz \right] dA$$

$$= \iint_D \left[yz \Big|_{z=0}^{1-x-y} \right] dA$$

$$= \iint_D y(1-x-y) - y(0) \; dA$$

$$= \iint_D (y - xy - y^2) \, dA$$

$$= \int_0^1 \int_0^{1-x} \left(y - xy - y^2 \right) dy\, dx$$

$$= \int_0^1 \frac{1}{2} y^2 - \frac{x}{2} y^2 - \frac{1}{3} y^3 \Big|_{y=0}^{1-x} dx$$

$$= \int_0^1 \left[\frac{1}{2}(1-x)^2 - \frac{x}{2}(1-x)^2 - \frac{1}{3}(1-x)^3 \right] dx$$

$$= \int_0^1 \left(\frac{1}{2} - x + \frac{1}{2} x^2\right) + \left(-\frac{x}{2} + x^2 - \frac{1}{2} x^3\right) + \left(\frac{1}{3} x^3 - x^2 + x - \frac{1}{3}\right) dx$$

$$= \int_0^1 \left(-\frac{x^3}{6} + \frac{x^2}{2} - \frac{x}{2} + \frac{1}{6} \right) dx$$

$$= \left(-\frac{x^4}{24} + \frac{x^3}{6} - \frac{x^2}{4} + \frac{x}{6} \right) \Big|_{x=0}^{1}$$

$$= \left(-\frac{1}{24} + \frac{1}{6} - \frac{1}{4} + \frac{1}{6} \right) - (0)$$

$$= \frac{-1 + 4 - 6 + 4}{24}$$

$$= \boxed{\frac{1}{24}}$$